

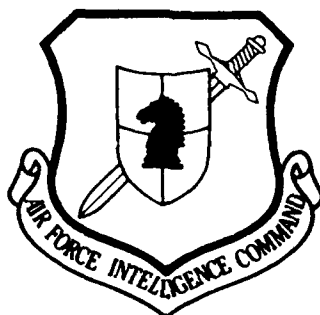
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SPECTRAL PROPERTIES OF PERIODICALLY
CORRELATED RANDOM PROCESSES

by

Ya. P. Dragan



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SPECTRAL PROPERTIES OF PERIODICALLY CORRELATED
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Block	Italic	Transliteration .	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
В в	<i>В в</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З з	<i>З з</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

*ye initially, after vowels, and after Ъ, Ь; e elsewhere.
When written as ѣ in Russian, transliterate as yě or ě.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	\sinh^{-1}
cos	cos	ch	cosh	arc ch	\cosh^{-1}
tg	tan	th	tanh	arc th	\tanh^{-1}
ctg	cot	cth	coth	arc cth	\coth^{-1}
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian English

rot curl
lg log

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SPECTRAL PROPERTIES OF PERIODICALLY CORRELATED RANDOM PROCESSES

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Submitted 20 Jun 70

In connection with the importance of the class of periodically correlated (PK) random processes (SP) in the investigation of rhythmic phenomena [1] and the insufficient degree of development of their theory, a need arises for a comprehensive study of them. In this article, developing the idea in work [2], the spectral properties of PKSP are investigated. By periodic correlation we understand the periodic nonstationarity of the second order.

As it follows from the results of work [2] and the definition of current spectrum [5, 6], the function of covariation of PKSP

is presented in the form

$$b(t, u) = r(t + u, t) = E \xi(t + u) \overline{\xi(t)}$$

$$b(t, u) = \int_{-\infty}^{\infty} e^{i u \lambda} d_{\lambda} F(\lambda, t), \quad (1)$$

in this case the dependence of spectral function on time is periodic, and this function possesses the expansion

$$F(\lambda, t) = \sum_{k=-\infty}^{\infty} F_k(\lambda) e^{i k \frac{2\pi}{T} t}, \quad (2)$$

where T - period of correlation of the process, and the functions $T_k(\cdot)$ are determined by the correlations

$$B_k(u) = \int_{-\infty}^{\infty} e^{i u \lambda} dF_k(\lambda), \quad b(t, u) = \sum_{k=-\infty}^{\infty} B_k(u) e^{i k \frac{2\pi}{T} t}. \quad (3)$$

Definition 1. As the precise period of a periodic function $f(\cdot)$ we shall call that least non-negative number T , for which at all t the equality $f(t+T) = f(t)$ is valid. The constant variable is a degenerate periodic function, since it is periodic in the case of any T and in the meaning of definition 1 it does not possess a precise period which is different from zero.

Definition 2. We will call the functions $B_k(\cdot)$ the correlation components (KK).

Taking into account that the dispersion of the values of the process

$$\sigma^2(t) = b(t, 0), \quad (4)$$

on the basis of what was said it is easy to be certain of the validity of the assertion *.

* In this article for simplicity we will assume that the mathematical expectation of the process is equal to zero. [end of footnote.]

Theorem 1. The current spectral function and the dispersion of the values of PKSP are periodic functions with a period equal to the period of correlation of the process. In principle the dispersion may be a precise period, equal to any part of the period of correlation, and it also may be degenerate.

The first assertion follows directly from formula (1) and the fact that T is the precise period of covariation. For proof of the second of formulas (3) and (4) we find

$$\sigma^2(t) = \sum_{k=-\infty}^{\infty} B_k(0) e^{ik \frac{2\pi}{T} t}. \quad (5)$$

Assume with a fixed integer m only the initial values of

$KK(B_{mp}(0) \neq 0, p = -\infty, \infty)$ will be different from zero. Then

$$\sigma^2(t+x) = \sum_{p=-\infty}^{\infty} B_{mp}(0) e^{ipm \frac{2\pi}{T} t} e^{ipm \frac{2\pi}{T} x},$$

from which, taking into account that $e^{i2\pi} = 1$, we see that the expression $x = \frac{T}{m}$ is the precise period of dispersion, i.e.

$\sigma^2\left(t + \frac{T}{m}\right) = \sigma^2(t)$. Dispersion is degenerate when $B_0(0) \neq 0$, and all the remaining $B_k(0) = 0$. These conditions are necessary and sufficient as a result of the uniqueness of the Fourier expansion.

Subsequently we will assume that PKSP is harmonizable (optionally absolutely in the meaning of Rozanov [4]) and its two-frequency spectral functional is differentiable in the sense of the theory of generalized functions, i.e.

$$r(s, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(s\lambda - t\mu)} d_\lambda d_\mu S(\lambda, \mu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(s\lambda - t\mu)} s(\lambda, \mu) d\lambda d\mu.$$

Then its derivative $s(\lambda, \mu)$ is a covariation of harmonics with frequencies λ and μ [6] and has the form

$$s(\lambda, \mu) = EZ'(\lambda) \overline{Z'(\mu)} = \sum_{k=-\infty}^{\infty} f_k(\lambda) \delta\left(\mu - \lambda + k \frac{2\pi}{T}\right). \quad (6)$$

where $f_k(\cdot) = F'_k(\cdot)$ and $Z'(\cdot)$ - generalized derivative in the sense of Ito [3] of the spectral measure of the process, entering into its representation

$$\xi(t) = \int_{-\infty}^{\infty} e^{it\lambda} dZ(\lambda) = \int_{-\infty}^{\infty} e^{it\lambda} Z'(\lambda) d\lambda. \quad (7)$$

From the representation of the function of covariation

$$b(t, u) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(uv - t\lambda)} f(v, \lambda) dv d\lambda, \quad (8)$$

is valid in the meaning of the theory of generalized functions for any harmonizable SP, if [2]

$$f(\mu - v, v) = s(v, \mu).$$

from formula (6) we find that the function of covariation of harmonics

$$f(\lambda, v) = \sum_{k=-\infty}^{\infty} f_k(\lambda) \delta\left(v + k \frac{2\pi}{T}\right). \quad (9)$$

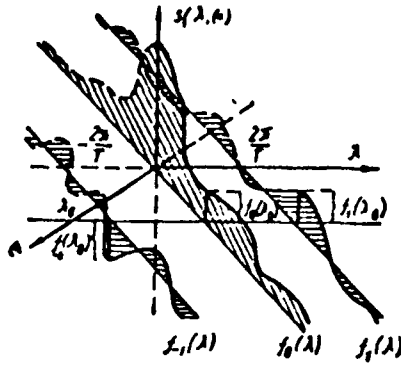


Figure 1.

Definition 3. We will call the functions $f_k(\cdot)$ the spectral components (SpK). Then from the last equality and formula (2) the validity of the theorem follows directly.

Theorem 2. The k -th spectral component and the k -th harmonic of change in the spectrum of PKSP coincide.

Theorem 3. Each harmonic with a frequency λ_0 of expansion (7) of the PKSP is correlated only with harmonics of frequencies

$\lambda_0 + k \frac{2\pi}{T}$, $k = -\infty, \infty$, and is not correlated with the remainder. The coefficient of correlation of the harmonics λ_0 and $\lambda_0 + k_0 \frac{2\pi}{T}$ is determined by the value $f_{k_0}(\lambda_0)$ of the complex-valued k_0 -th SpK $f_{k_0}(\cdot)$ (Fig. 1).

For establishing the bond between the spectral components and the positive and negative indexes we will use the correlation [2] for KK:

$$B_k(u) = \overline{B_{-k}(-u)} e^{ik \frac{2\pi}{T} u}.$$

Then, taking the first of formulas (3) into account, we find

$$f_{-k}(\lambda) = \overline{f_k\left(\lambda + k \frac{2\pi}{T}\right)}.$$

The sets of frequencies $\lambda_k = \left\{ \lambda_0 + k \frac{2\pi}{T}, k = -\infty, \infty \right\}$ do not form partitioning of the axis of frequencies into classes of equivalence, since the magnitude of correlation depends not only on k , but also on λ_0 .

Partitioning will take place only in the case of PK of white noise, defined (see below) as a process with covariation $r(s, t) = g(t) \delta(s-t)$,

where $g(t) = \sum_{k=-\infty}^{\infty} c_k e^{i k \frac{2\pi}{T} t} \geq 0$ - periodic function, since for it in formula (6) $f_k(\lambda) c_k$, and it possesses stationarily correlated harmonics, i.e. $s(\lambda, \mu) = s(\lambda - \mu)$.

Let us note the validity of the general theorem.

Theorem 4. We will present the covariation of any process which is stationarily correlated in spectrum in the form of the product $g(t) \delta(s-t)$, where $g(t) \geq 0$, and conversely, any nonstationary noise is a harmonizable process with stationarily correlated harmonics.

The last assertion follows directly from the theorem of collation of spectra and the Fourier-presentation of the δ -function. Then

$$r(s, t) = g(t) \delta(t-s) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(s\lambda - t\mu)} G(\lambda - \mu) d\lambda d\mu$$

and $G(\cdot)$ - Fourier-image of the function $g(\cdot)$.

Since the function

$$B_0(u) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T b(t, u) dt = \frac{1}{T} \int_0^T b(t, u) dt \quad (10)$$

exists and is defined positively, then $f_0(\lambda) \geq 0$ and it describes the powers of the harmonics of the components of the process. On the other hand, the function, determined by the first of equalities (10), is called the covariation of a stationary approximation to a nonstationary process. Everything taken together shows that the theorem is valid.

Theorem 5. A stationary approximation to PKSP - this is a stationary process consisting of the same harmonic components and with the same powers as PKSP, but that are not correlated. Averaging (10) is equivalent to elimination of the correlation of harmonics, and the power $B_0(0)$ of stationary approximation is equal to the power of the PKSP which is average for the period.

Actually, from formulas (10) and (4) it is evident that

$$B_0(0) = \frac{1}{T} \int_0^T \sigma^2(t) dt. \quad \text{In that particular case when } b(t, \cdot) \text{ is almost a}$$

periodic function, functions $B_k(\cdot)$ will be almost periodic, and if

$\{\lambda_l, l = -\infty, \infty\}$ - set of indices of a Fourier function $b(t, \cdot)$, then the function of the frequency covariation

$$s(\lambda, \mu) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} c_{kl} \delta\left(\lambda - \left[k \frac{2\pi}{T} - \lambda_l\right]\right) \delta(\mu - \lambda_l) \quad (11)$$

is different from zero only in the points of intersection of the set

$\left\{\lambda = k \frac{2\pi}{T} - \lambda_l\right\}$ with the set $\{\mu = \lambda_l\}$. It follows from here that only the harmonics, for which $\lambda_l = k \frac{2\pi}{T}$, will be correlated. In

particular, for periodic processes [2,6] $\lambda_l = \frac{2\pi}{T}$, and all the harmonics are correlated, and the spectrum is averaged in points

$$\left(k \frac{2\pi}{T}, l \frac{2\pi}{T}\right), k, l = -\infty, \infty.$$

In a general case the following theorem takes place.

Theorem 6. If the function $b(t, \cdot)$ is periodic, then its period is a multiple of the period of correlation.

Assume the period of the function $b(t, \cdot)$ is equal to T_0 . Then on the basis of formula (3) the function possesses a Fourier expansion

$$b(t, u) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} b_{kl} e^{i2\pi \left(k \frac{t}{T} + l \frac{u}{T_0}\right)},$$

from which it is easy to see that all the KK will be periodic with the period T_0 :

$$B_k(u) = \sum_{l=-\infty}^{\infty} b_{kl} e^{i2\pi \frac{l}{T_0} u},$$

and in this case in conformity with formula (9) $f(v, \lambda) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} b_{kl} \delta\left(\lambda - l \frac{2\pi}{T_0}\right) \times$

$\times \delta\left(v - k \frac{2\pi}{T}\right)$. From this expression and the identity which is known

from the theory of generalized functions $0 \cdot \delta(\cdot) \equiv 0$ it follows that

when $\left\{l \frac{2\pi}{T_0}, l = -\infty, \infty\right\} \cap \left\{k \frac{2\pi}{T}, k = -\infty, \infty\right\} = \emptyset$, then $f(v, \lambda) \equiv 0$, and

then on the basis of the uniqueness of the Fourier transform from formula (8) we have the identity $b(t, u) \equiv 0$, i.e., the PKSP is

degenerated to zero. In the case of non-degeneration the inclusion of

$\left\{k \frac{2\pi}{T}\right\} \subset \left\{l \frac{2\pi}{T_0}\right\}$ takes place, since T is the precise period of correlation. Since both these sets of points form arithmetic progressions with the differences $\frac{2\pi}{T}$ and $\frac{2\pi}{T_0}$ respectively, then there exists such a natural number p that $\frac{2\pi}{T} = p \frac{2\pi}{T_0}$. The latter is equivalent to the equality $T_0 = pT$, proving the theorem.

Subsequently it will be convenient to use the following definitions.

Definition 4. In a general representation of PKSP [2]

$$\xi(t) = \sum_{k=-\infty}^{\infty} \xi_k(t) e^{ik \frac{2\pi}{T} t}, \quad (12)$$

where $\{\xi_k(\cdot)\}$ - stationary stationarily bound processes (stationary components (SK PKSP)).

Definition 5. We will designate as periodically correlated white noise a PKSP for which in formula (6) $f_k(\lambda) = f_k$ for all λ and k .

From definition 5 it is evident that

$$r(s, t) = 2\pi \sum_{k=-\infty}^{\infty} f_k e^{ik \frac{2\pi}{T} t} \delta(s - t) = g(t) \delta(s - t),$$

i.e., for PK of white noise $b(t, u) = g(t) \sigma(u)$, where $g(\cdot) \geq 0$ - periodic function of the period, equal to the period of correlation.

Theorem 7. If SK PKSP - white noises, then it degenerates into PK white noise.

Actually, using formula [2]

$$b(t, u) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} r_{l, l-k}(u) e^{i \frac{2\pi}{T} (kt - lu)} \quad (13)$$

and the condition of noncorrelation of SK $r_{i, j}(u) = A_{i, j} \delta(u)$, and also the property $f(x) \cdot \delta(x) = f(0) \delta(x)$, we obtain

$$b(t, u) = \sum_{k=-\infty}^{\infty} \left[\sum_{l=-\infty}^{\infty} A_{l, l-k} \right] e^{i u \frac{2\pi}{T} t} \delta(u) = g(t) \delta(u),$$

where $g(\cdot)$ - periodic function of period T . It is non-negative when $A_{1,j} \geq 0$.

Theorem 8. If SK are uncorrelated, then PKSP degenerates into a stationary SP, and if they degenerate into random variables, then the process will be periodic, if these variables are uncorrelated the process will be periodic stationary [6].

Proof follows from formulas (9) and (10).

Definition 6. We will designate as periodic white noise a process for which in formula (11) $c_{kl} = c_k$ when $\lambda_l = l \frac{2\pi}{T}$. Then its covariation

$$r(s, t) = \sum_{k=-\infty}^{\infty} c_k e^{i k \frac{2\pi}{T} t} \sum_{l=-\infty}^{\infty} e^{i l \frac{2\pi}{T} (t-s)},$$

which taking into account the Poisson formula from the theory of generalized functions

$$\sum_{l=-\infty}^{\infty} \delta(x - lT) = \frac{1}{T} \sum_{l=-\infty}^{\infty} e^{i l \frac{2\pi}{T} x},$$

giving a representation of the periodic δ -function $\delta_T(x) = \sum_{l=-\infty}^{\infty} \delta(x - lT)$,

can be written in the form $r(s, t) = Tg(t) \delta_T(t-s)$, which proves the statement.

Theorem 9. Periodic noise does not coincide with PK noise.

The following theorems are important for an understanding of the structure of PKSP.

Theorem 10. The state of being correlated by stationary PKSP components and the state of being correlated by its harmonic components are equivalent.

Since SK are correlated, and by definition 4 they are stationarily bound, then their mutual covariation permits the representation

$$r_{1,j}(u) = \int_{-\infty}^{\infty} e^{i u \lambda} dS_{1,j}(\lambda).$$

where $EdZ_l(\lambda) \overline{dZ_l(\mu)} = dS_{l,l}(\lambda) \delta(\lambda - \mu) d\mu$, and $Z_1(\cdot)$ - random measure, corresponding to the process $\xi_L(\cdot)$ in expansion (7). Then from formulas (3) and (13) we find

$$B_k(u) = \sum_{l=-\infty}^{\infty} r_{l,l-k}(u) e^{u \frac{2\pi}{T} l},$$

from which

$$F_k(\lambda) = \sum_{l=-\infty}^{\infty} S_{l,l-k} \left(\lambda - l \frac{2\pi}{T} \right),$$

from which the validity of the theorem follows directly, i.e., the correlated state of the harmonics implies the correlated state of stationary components, and vice versa. In particular, from here the spectral function of the stationary approximation to PKSP

$$F_0(\lambda) = \sum_{l=-\infty}^{\infty} S_{l,l} \left(\lambda - l \frac{2\pi}{T} \right), \text{ i.e., is equal to the sum of the displaced}$$

spectral functions of stationary components, and the shift is equal to the frequency of the correlated state of PKSP multiplied by the SK number.

Theorem 11. An invariant ideal band-pass filter separates from the PKSP its uncorrelated harmonic components when, and only when, the width of its bandwidth is less than the frequency of the correlated state of PKSP.

Proof follows from the fact that the spectral density of the transform of the harmonized process with the help of the invariant operator $\hat{\Phi}$ with the frequency characteristic $\Phi(\lambda)$ is expressed through the spectral density of the process by the correlation [2,6]

$$s_{\hat{\Phi}}(\lambda, \mu) = \Phi(\lambda) \overline{\Phi(\mu)} s_{\xi}(\lambda, \mu),$$

which taking formula (6) into account takes the form

$$s_{\hat{\Phi}}(\lambda, \mu) = \Phi(\lambda) \sum_{k=-\infty}^{\infty} f_k(\lambda) \overline{\Phi \left(\lambda - k \frac{2\pi}{T} \right)} \delta \left(\mu - \lambda + k \frac{2\pi}{T} \right).$$

Assume now that the frequency characteristic of the filter $\Phi_{\lambda_0}(\lambda)$ is equal to zero outside of the interval $I_{\lambda_0} = \left[\lambda_0 - \alpha \frac{\pi}{T}, \lambda_0 + \alpha \frac{\pi}{T} \right] (\alpha < 1)$,

i.e., $\Phi_{\lambda_0}(\lambda) = \Phi_{\lambda_0}(\lambda) \chi_{I_{\lambda_0}}(\lambda)$, where $\chi_{\lambda}(\cdot)$ - characteristic function of the

set A . Then

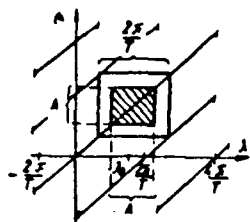


Figure 2.

$$s_{\hat{\Phi}_k}(\lambda, \mu) = \Phi_{\lambda_0}(\lambda) \overline{\Phi_{\lambda_0}(\mu)} \times \\ \times \sum_{k=-\infty}^{\infty} f_k(\lambda) \chi_{\lambda_0}(\lambda) \chi_{\lambda_0}(\mu) \delta\left(\mu - \lambda + k \frac{2\pi}{T}\right).$$

Considering the property of the δ -function $\chi_A(x) \cdot f(x) \delta(x-a) = \chi_A(a) f(a)$, we see that the product of the last three factors differs from zero only in the case when $|\mu - \lambda| < \frac{2\pi}{T}$, and as a result of the independence of λ and μ from the inequalities

$|\mu - \lambda| \leq |\mu| + |\lambda|$ it turns out that the product is different from zero only when $|\mu| \leq \frac{\pi}{T}$ and $|\lambda| \leq \frac{\pi}{T}$. Under these conditions the mentioned product is equal to $\delta_{k,0} \delta(\lambda - \mu)$, and then

$$s_{\hat{\Phi}_k}(\lambda, \mu) = \chi_{\lambda_0}(\lambda) f_0(\lambda) |\Phi_{\lambda_0}(\lambda)|^2 \delta(\mu - \lambda).$$

From here the correlation function of the transform

$$r_{\hat{\Phi}_k}(s, t) = \int_{\lambda_0} e^{i(s-t)\lambda} f_0(\lambda) |\Phi_{\lambda_0}(\lambda)|^2 d\lambda$$

i.e., the filtration of the PKSP with a bandwidth less than the frequency of its correlated state is equivalent to the filtration of its stationary approximation. The sufficiency of the condition of the theorem follows from the fact that if $A = [\lambda_0 - \Delta, \lambda_0 + \Delta]$ is designated, then

$$\chi_A(\lambda) \chi_A(\mu) \delta\left(\lambda - \mu + k \frac{2\pi}{T}\right) = \chi_A(\lambda) \delta(\lambda - \mu) \delta_{k,0}$$

only in the case when the set $A \times A$ does not intersect with one of the straight lines $\mu = \lambda - k \frac{2\pi}{T}$, $k = -\infty, \infty$, with the exception of the diagonal $\mu = \lambda$ (Figure 2).

Another proof of this theorem can be obtained if in expansion (12) of the PKSP each stationary component is replaced by its Cramer - Kolmogorov expansion

$$\xi_k(t) = \int_{-\infty}^{\infty} e^{i\omega t} d\zeta_k(\omega).$$

Then the harmonic expansion (7) for PKSP takes the form

$$\xi(t) = \int_{-\infty}^{\infty} e^{i\lambda t} d \left[\sum_{k=-\infty}^{\infty} \zeta_k \left(\lambda - k \frac{2\pi}{T} \right) \right],$$

from where follows directly the type of correlated state of the harmonics mentioned in theorem 3:

$$\begin{aligned} s(\lambda, \mu) &= \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} E \zeta'_k \left(\lambda - k \frac{2\pi}{T} \right) \zeta'_l \left(\mu - l \frac{2\pi}{T} \right) = \\ &= \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} s_{k, k-n} \left(\lambda - k \frac{2\pi}{T} \right) \delta \left(\mu - \lambda + n \frac{2\pi}{T} \right). \end{aligned}$$

Thus, in the representation of PKSP of type (7)

$$Z(\lambda) = \sum_{k=-\infty}^{\infty} \zeta_k \left(\lambda - k \frac{2\pi}{T} \right)$$

and in the sense of the theorem of generalized functions

$$f(v, \lambda) = E Z'(v) \overline{Z'(\nu + \lambda)}.$$

Definition 7. We will call a harmonizable SP degenerate if its dispersion is constant.

According to theorem 1 such a process may be nonstationary.

Theorem 12. A non-degenerate harmonizable SP with periodic dispersion belongs to the class of almost periodically correlated.

For proof of this assertion from formula (8) we determine the dispersion

$$\sigma^2(t) = \int_{-\infty}^{\infty} e^{-i\lambda t} \int_{-\infty}^{\infty} f(v, \lambda) dv d\lambda. \quad (14)$$

It is evident from expression (14) that dispersion can be periodic with a precise period T_0 only when

$$\int_{-\infty}^{\infty} f(v, \lambda) dv = \sum_{l=-\infty}^{\infty} c_l \delta \left(\lambda + l \frac{2\pi}{T_0} \right).$$

The latter in turn is possible only when

$$f(v, \lambda) = f_0(v, \lambda) + f_1(v, \lambda) \sum_{n=-\infty}^{\infty} \delta(\lambda - \lambda_n) = f_0(v, \lambda) + \sum_{n=-\infty}^{\infty} f_n(v) \delta(\lambda - \lambda_n),$$

where $f_n(v) = f_1(v, \lambda_n)$. In this case the following conditions should be fulfilled.

$$\int_{-\infty}^{\infty} f_n(v) dv = \begin{cases} c_l & \text{when } \lambda_n = l \frac{2\pi}{T_0}; \\ 0 & \text{in the opposite case,} \end{cases}$$

$$\int_{-\infty}^{\infty} f_0(v, \lambda) dv = 0.$$

From the last equality and formula (14) it follows that dispersion of the component of a random process, possessing a continuous spectrum $f_0(v, \lambda)$, is equal to zero. Since the process is centered, then this component is identically equal to zero, which also proves the theorem.

It follows from expression (14) that if z is taken as y_0 , then the inequality (12) will be fulfilled. For a more rapid convergence of formula (11) when $y \gg 1$ it is possible to use as the initial approximation the magnitude

$$z_1 = \sqrt{z^2 - \ln \frac{z\sqrt{\pi}}{2}}. \quad (15)$$

Table

(1) Исходные данные		Абсолютная ошибка формул (2)		
		(11)	(16)	(6)
$x \in [0; 0.9999]$	1	8×10^{-2}	5×10^{-2}	0.16
$y \in [0; 2.751]$	2	2×10^{-2}	8×10^{-4}	0.15
$y_0 = z$	3	7×10^{-4}	5×10^{-9}	0.1
$ \Delta_0 < 0.17; \Delta_1 < 0.46$	4	2×10^{-6}	10^{-21}	0.02
$x \in [0; 1 - 10^{-12}]$	1	10^{-2}	6×10^{-4}	3×10^{-3}
$y \in [0; 5.042]$	2	10^{-4}	10^{-10}	10^{-14}
$y_0 = z_1$	3	10^{-8}	2×10^{-31}	3×10^{-58}
$ \Delta_0 < 0.1; \Delta_1 < 0.095$				

Key: (1) Initial data; (2) Absolute error of formulas.

The table gives data on the absolute error of formulas (6) and (11), and also of the iteration formula of the third order

$$y_{i+1} = y_i - \frac{\sqrt{\pi}}{2} [\Phi(y_i) - x] e^{y_i^2} + \frac{\pi}{4} y_i [\Phi(y_i) - x] e^{2y_i^2} \quad (16)$$

in the case of the initial approximation (13) and

$$y_0 = z_2 = \begin{cases} z & \text{when } 0 < x < 0.82; \\ z_1 & \text{when } x > 0.82. \end{cases} \quad (17)$$

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